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ECE 3712 – Introduction to Electromagnetic Fields and Waves

**Project 1**

**Introduction:**

Electrostatics is the study of stationary electric charges and fields. It is important to learn how a conductor with a specific charge affects the space around it by the means of electric potential and electric field intensity, this can be more clearly seen when a close examination is done to the technology that it is used daily. For example, applications of electrostatics exist in photocopiers, laser printers, ink jet printers, and electrostatic air filters. More specifically, an ink jet printer which is mostly used to print computer-generated text employs a nozzle that makes a spray of ink droplets, which are given an electrostatic charge. As soon as they are charged, the droplets are directed using pairs of charged plates that are built to precisely form specific characters on paper. It is possible to see from the previous example in addition to the many other technologies, that electrostatics plays an important role is our everyday lives. Without electrostatic forces, it would be much more difficult to control or direct small particles that can serve useful purposes.

**Problem Statement:**

The givens for this problem is a grid with 2 conductors in the middle of quadrant 1 and quadrant 3, with a charge of 10 volts and -10 volts respectively. The grid is a square of 1 unit wide centered at the origin, where the voltage at the boundaries is zero volts. The purpose of this project is to find the electric potential and the electric field intensity at all grid points in the domain, draw the equipotential lines in the domain, draw the contour plot of constant field intensity, and find the stored energy per unit volume.

**Basic Concepts:**

To solve the given problem, it is first necessary to discover how to measure the electric potential at each node in the grid. First, it is important to consider the difference equation that approximates the Laplace equation at any grid point ( i , j ). The following shows the difference equation:

|  |  |
| --- | --- |
| $$V\_{i,j}=\frac{V\_{i+1,j}+V\_{i-1,j}+V\_{i,j+1}+V\_{i,j-1}}{4}$$ | Difference Equation (1) |

Based on the difference equation it is possible to see that the electric potential at each node located in ( i , j ) is equal to the average of the electric potential at all four adjacent nodes in the grid. Before proceeding it is important to note that to the difference equation was obtained by ignoring all higher order terms during the Taylor series expansion, therefore it can only give an approximate solution. However, the solution can be much more accurate by discretizing the domain to a finer grid in which Δx and Δy are small. Due to this, many more equations must be solved simultaneously and demand more computer time and memory.

The algorithm created to solve for the electric potential at each grid point begins by first displacing the grid plane where the conductors are located into the first quadrant and scaling the plane to go from 0 to 40 units from side to side. Then, when the conductors have been placed in the corresponding location and size in the first quadrant, a matrix will be created to model the plane where the conductors are located that will hold 10 volts for the first charge in the top right of the first quadrant and -10 volts for the second charge in the bottom left of the first quadrant. It is important to keep in mind, that the conductors were squares of width 0.1 but due to the scaling, they are now 4 units wide. Then, all other points in the matrix that are not part of the conductor will be equal to zero. Now that the matrix that models each node in the grid is ready, the next step is to build an algorithm that will calculate the electric potential at each node in the grid many times to make the numbers converge close to the real value. This algorithm will specifically update the electric potential at each node in the grid 2000 times to make sure the number converges close to the real value. The last step needed to fully build an algorithm that will be able to iterate the calculation 2000 times is to model the equation. This is done by doing 2 nested for loops. The inner for loop will take care of changing the position of the node in the y direction, and the outer for loop will take care of changing the position of the node in the x direction. Inside of the 2 nested for loops, the average electric potential of the 4 adjacent nodes will be calculated. Overall, the algorithm will calculate the electric potential node by node inside of the matrix starting at element (2,2) going down to the column and every time it finishes a column it will move to the next column to the right until the whole matrix is done.

To calculate the electric field intensity, the following formula is used

|  |  |
| --- | --- |
|  | Equation 2 |

In MATLAB there is a function that does exactly that when the input is the matrix of electric potential and the spacing between the nodes. The function is called “gradient”.

Now that the electric potential and electric field intensity is known for each node in the grid, it is now possible to draw the equipotential lines in the domain, as well as the contour plot of constant filed intensity. Using MATLAB, the functions called “contour” and “quiver” were used to draw the equipotential lines and contour plot along with the corresponding matrix of electric potential or electric field intensity as input.

To find the stored energy per unit volume the following equation was used:

|  |  |
| --- | --- |
|  | Equation 3 |
|  |  |

In equation 3, εr is equal to 100, εo is equal to 8.85\*10^-12, and E is the electric field intensity that was calculated before. Now, to calculate the integral in MALTLAB, a for loop must be done to add all the electric field intensity at each element or node in the plane. Then, the sum of all those values is multiplied by the constants seen in equation 3 to obtain the stored energy per unit volume.

To find the displacement current, conduction current, and total current the following equation was used:

|  |  |
| --- | --- |
|  | Equation 4 |

According to equation 4 above, conduction current is equal to the integral of the current density, and displacement current is equal to the derivative with respect to time of the integral of the electric displacement. Now, to specifically calculate the conduction current, the first thing is to find current density. The equation for the current density is shown below, where sigma is equal to 2\*10^-4:

|  |  |
| --- | --- |
|  | Equation 5 |

It is possible to see that the current density can be easily calculated by multiplying the electric field intensity at each node and multiplying the value by sigma. Then, after the current density is obtained, a for loop is used in MATLAB to add the current densities at each node to obtain the conduction current.

To find the displacement current, it is first necessary to multiply all the elements of the electric field intensity by εo since the electric displacement is equal to εo multiplied by the electric field intensity. Once again, a for loop is used to integrate the electric displacement for each element in the matrix. At last, the partial derivative with respect to time is calculated on the result of the integration by using a MATLAB function called “diff”. Then, to calculate the total current based on an input of AC voltage the following equation was used in MATLAB:

|  |  |
| --- | --- |
|  | Equation 6 |

In equation 6, Inot is equal to the addition of the conduction current and the displacement current, and omega is equal to 1kHz multiplied by 2\*PI.

**Results:**

After writing the code in MATLAB and explaining how the code worked in the Basic Concepts section of this report, the following results were obtained:



Figure 1: Equipotential lines in the Domain. This graph was created by plotting the electric potential in the given domain.

According to figure 1 above, it is possible to see the Equipotential lines created from the 2 conductors that were given. By using reason it is possible to see that the equipotential lines make sense because at nodes close to the top conductor, the voltage is going to be close to 10 volts, at nodes close the bottom conductor, the voltage is going to be close to -10 volts, and the line that divides the 2 conductors and makes a symmetric image will have a voltage of zero because the voltage from each conductor will cancel out.



Figure 2: Contour Plot of Constant Field Intensity. This graph was created by plotting the electric field intensity in the domain.

Next is the contour plot of constant field intensity that was calculated based on the electric field intensity at each node. It is possible to see that the image makes sense because it is expected that the positive charged conductor will repel everything around it and the negative charged conductor will attract everything around it. Also, the closer the nodes are to the conductor, the stronger the repulsive and attractive forces which is why the arrows near the conductors are bigger and the arrows far away from the conductor. It is also important to note that the arrows are perpendicular to the equipotential lines shown in Figure 1.

The stored energy per unit volume was determined to be approximately 0.43 joules, the value makes sense because the size of the conductors were very small and the domain itself was small which gives a small energy per unit volume. The number does seem reasonable.

As for the conduction current, and displacement current the values were determined to be approximately 6.22 Amps, and -5.3\*10^-10 correspondingly. The value for the conduction current seem a little bit too high while the displacement current seems low, but they make sense because the conductors affect the space around them to create a current.

At last, the total current, was determined to be equation 6, where Inot is equal to the sum of the conduction current and the displacement current, and omega is equal to 1kHz time 2\*PI. This makes sense because the total current will vary with time because the voltage is AC.

**Conclusions:**

It is very useful to see that the arrows of the contour plot are perpendicular to the equipotential lines because it makes it easier to conceptualize the relationship between the electric potential and the electric field intensity. This project was successful because it provided reasonable value for the electric potential, electric field intensity, stored energy per unit volume, conduction current, displacement current, and total current. This project made it easier to understand the concepts by applying them to a real-life problem.